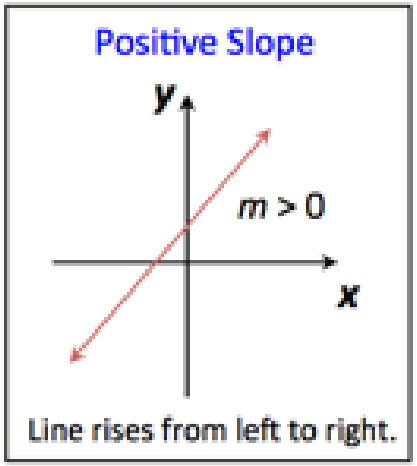
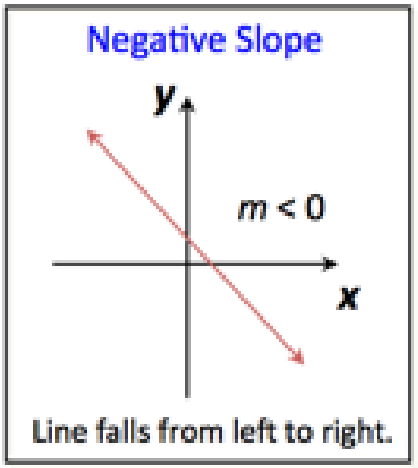
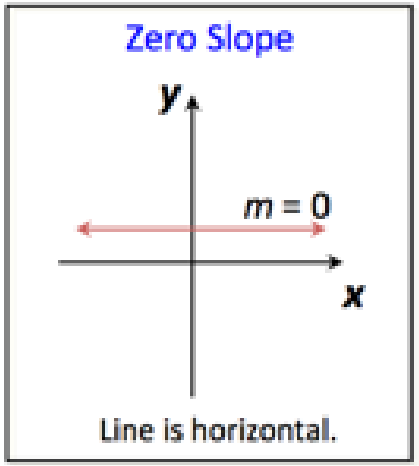
# 4.3 Find Slope and Rate of Change

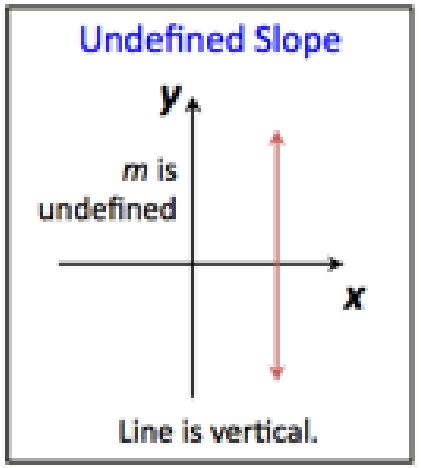
Slope: is the ratio of the **rise to the run** over 2 points of a non-vertical line.

## The Possibilities for a Line's Slope



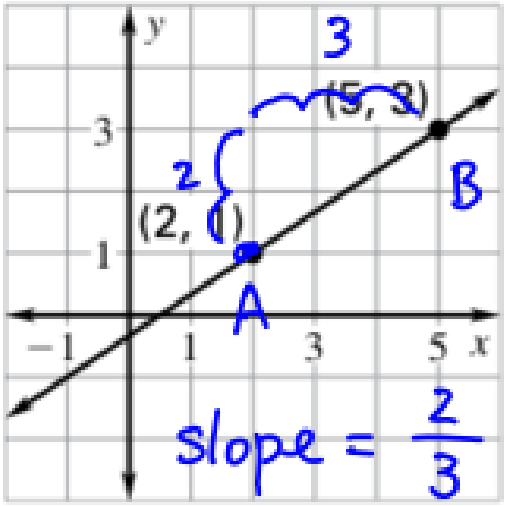



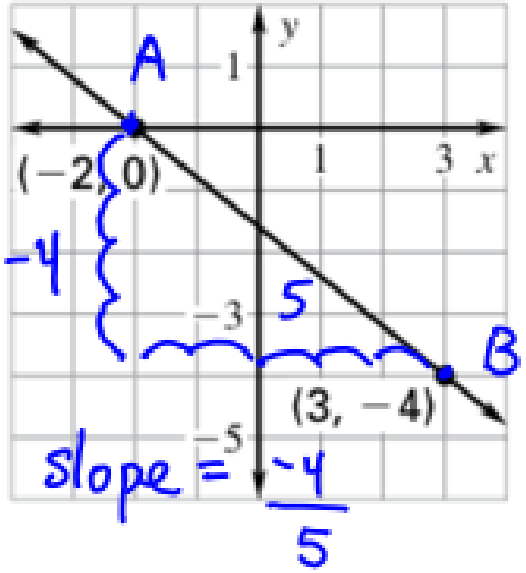



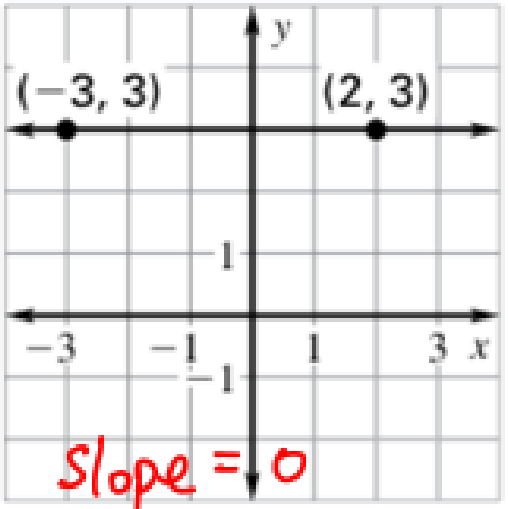



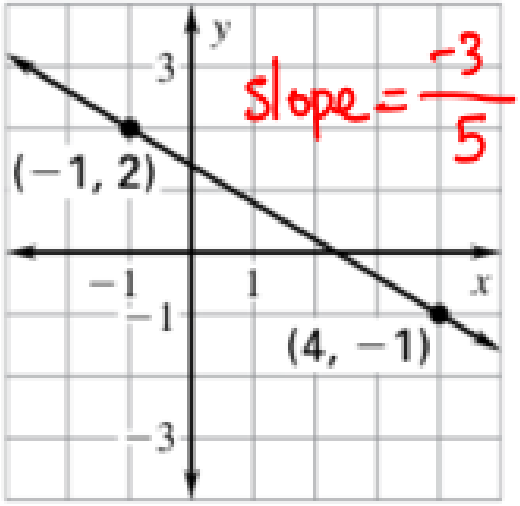
**Ex1) Find the slope:** $\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{+\text{up} \; ; \; -\text{down}}{+\text{right} \; ; \; -\text{left}}$

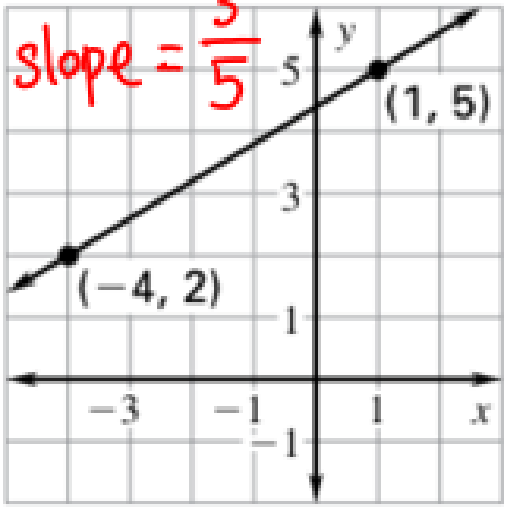
Slope: is the ratio of the **rise to the run** over 2 points of a non-vertical line.

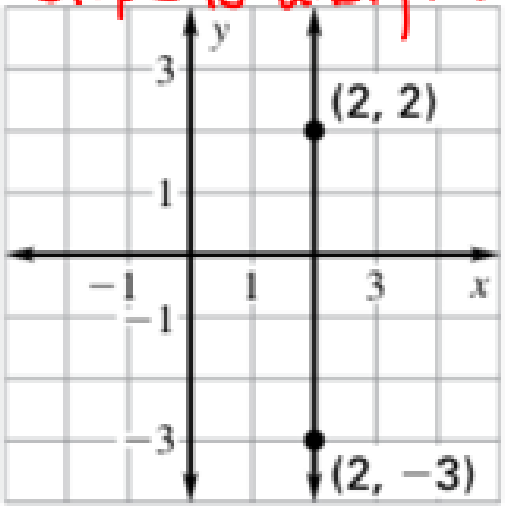


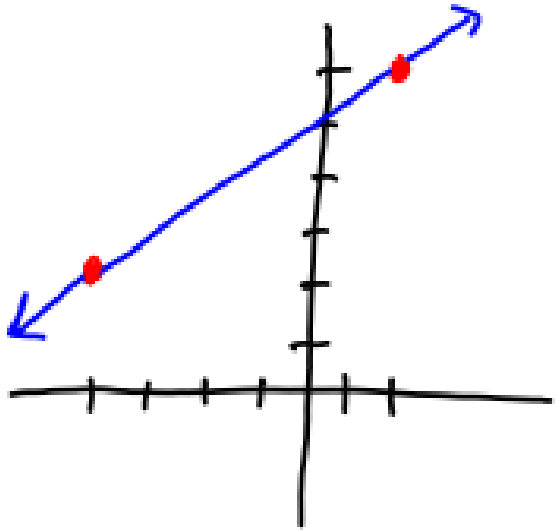
# Finding Slope between two given points:

**Slope Formula:** $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

**Ex2) Find the slope of: (−4, 2) and (2, 6)**

$x_1\ y_1 \qquad x_2\ y_2$

$$m = \frac{6-2}{2+(+4)} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

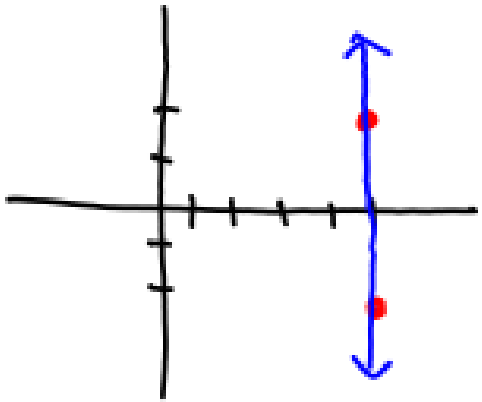


**Ex3) Find the slope of: (5, 2) and (5, –2)**

$x_1\ y_1 \qquad x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{5 - 5} = \frac{-4}{0}$$

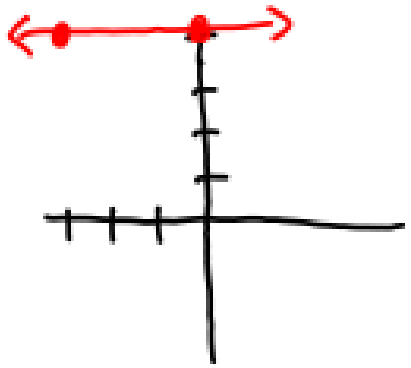
undefined



**Ex4) Find the slope of: (0, 4) and (–3, 4)**

$x_1\ y_1 \qquad x_2\ y_2$

$$m = \frac{4 - 4}{-3 - 0} = \frac{0}{-3} = 0$$



**Try) Find the slope of the line that passes through the points.**

1. (5, 2) and (4, –1)

$x_1\ y_1 \qquad x_2\ y_2$

$$m = \frac{-1 - 2}{4 - 5}$$

$$= \frac{-3}{-1}$$

$$= \boxed{3}$$

2. (–2, 3) and (4, 6)

$$m = \frac{6 - 3}{4 - (-2)}$$

$$= \frac{3}{6}$$

$$= \boxed{\frac{1}{2}}$$

**Ex5) The table shows the cost of using a computer at an Internet cafe for a given amount of time. Find the rate of change in cost with respect to time.**

| | | | | |
|---|---|---|---|---|
| x | Time (hours) | 2 | 4 | 6 |
| y | Cost (dollars) | 7 | 14 | 21 |

+2 +2

+7 +7

rate of change

$= \text{slope} = \dfrac{\text{changes of } y}{\text{changes of } x}$

$ROC = \dfrac{7}{2} = 3.5$

The rate of change is $3.50 per hour.

**Try) The table shows the distance a person walks for exercise. Find the rate of change in distance with respect to time.**

| Time (minutes) x | Distance (miles) y |
|---|---|
| 30 | 1.5 |
| 60 | 3 |
| 90 | 4.5 |

+30, +30 (time); +1.5, +1.5 (distance)

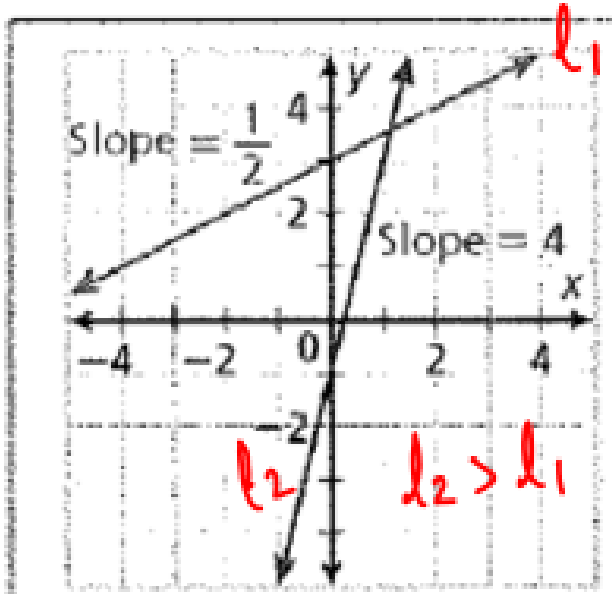
$$ROC = \frac{\text{change of } y}{\text{change of } x}$$

$$= \frac{1.5}{30} = 0.05$$

The rate of change is 0.05 miles per minute.

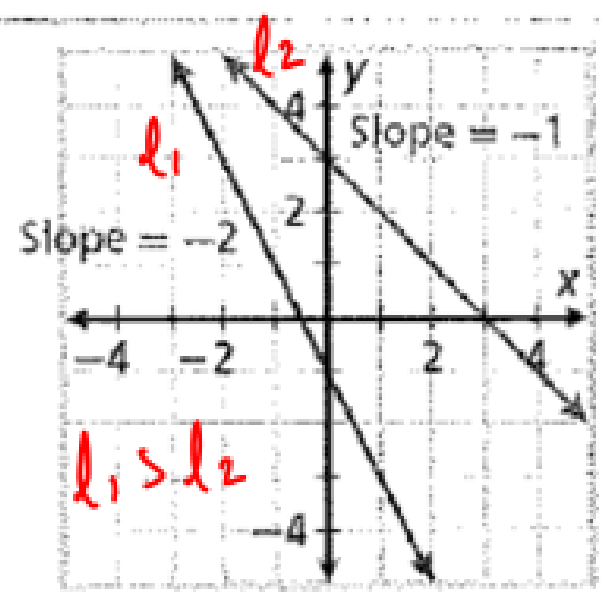
# Comparing Slopes

A line's slope is measure of its **steepness**.



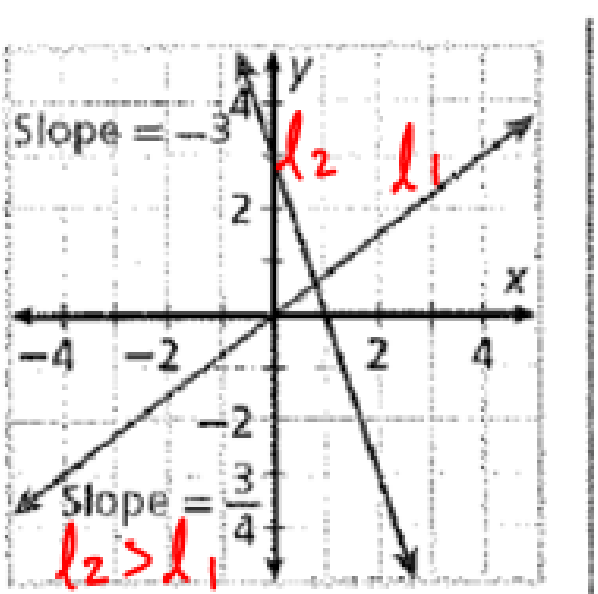

The line with slope 4 is steeper than the line with slope $\frac{1}{2}$.

$$|4| > \left|\frac{1}{2}\right|$$




The line with slope −2 is steeper than the line with slope −1.

$$|-2| > |-1|$$




The line with slope −3 is steeper than the line with slope $\frac{3}{4}$.

$$|-3| > \left|\frac{3}{4}\right|$$

# Finding Slope from an Equation

**Standard Form:** $Ax + By = C$ (A must be positive)

**Slope-intercept Form:** $y = mx + b$

↑ slope ↑ y-int

Graph the Equation:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{+up ; -down}}{\text{right}}$$

Ex6) Find the slope and y-intercept of the line described by the equation. $y = mx + b$ (get y alone)

a) $-2x - 5y = 10$

$+2x \qquad +2x$

$\frac{-5y}{-5} = \frac{2x}{-5} + \frac{10}{-5}$

$y = -\frac{2}{5}x - 2$

Slope = $-\frac{2}{5}$

y-int : $(0, -2)$

b) $4x + 2y = 8$

$-4x \qquad -4x$

$\frac{2y}{2} = \frac{-4x}{2} + \frac{8}{2}$

$y = -2x + 4$

slope = $-2$

y-int : $(0, 4)$

c) $-6x+2y=5$

+6x +6x

$\frac{2y}{2} = \frac{6x}{2} + \frac{5}{2}$

$y = 3x + \frac{5}{2}$

Slope = 3

y-int : $(0, \frac{5}{2})$

d) $\frac{1}{3}x - \frac{2}{4}y = 1$ get common denominator

$\frac{4}{12}x - \frac{6}{12}y = \frac{12}{12}$

$4x - 6y = 12$

$-4x \quad -4x$

$\frac{-6y}{-6} = \frac{-4x}{-6} + \frac{12}{-6}$

Slope = $\frac{2}{3}$

y-int : $(0, -2)$

$y = \frac{2}{3}x - 2$

e) $\frac{2y}{2} = \frac{8}{2}$

$y = 4$

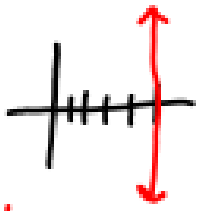
No x-int

Horizontal

Slope = 0

y-int $(0, 4)$

f) $3x - 7 = 8$



No y-int ; Vertical

Slope is undefined

$x = 5$

Try) Find the slope and y-intercept of the line described by the equation.

1) $-4x+8y=32$

$8y = 4x + 32$

$y = \frac{1}{2}x + 4$

Slope $= \frac{1}{2}$

y-int : $(0, 4)$

2) $8x-6y=24$

$-6y = -8x + 24$

$y = \frac{4}{3}x - 4$

Slope $= \frac{4}{3}$

y-int : $(0, -4)$

3) $-5x+2y=3$

$2y = 5x + 3$

$y = \frac{5}{2}x + \frac{3}{2}$

slope $= \frac{5}{2}$

y-int : $(0, \frac{3}{2})$