5.1 Solving System of Equations by Graphing

System of Equations: Consists of **2 or more** linear **equations** in the same variables.

Example: $\begin{cases} -3x + y = -7 \\ 2x + 2y = 10 \end{cases}$ The solution is an order pair: a coordinate : (x, y)

Check which ordered pair is a solution? (3, 2) or ~~(2, -1)~~

put each point into both equation.

Check : $(3, 2)$
      x y

1) $-3(3) + 2 \overset{?}{=} -7$ ✓

2) $2(3) + 2(2) \overset{?}{=} 10$ ✓

$\boxed{(3, 2) \text{ is the Solution}}$

Check : $(2, -1)$
           x y

1) $-3(2) + (-1) \overset{?}{=} -7$ ✓

2) $2(2) + 2(-1) \overset{?}{=} 10$ ✗

Ex1) Solve the systems equation by graphing.

Solution: the point (x, y) where the two lines intercept.
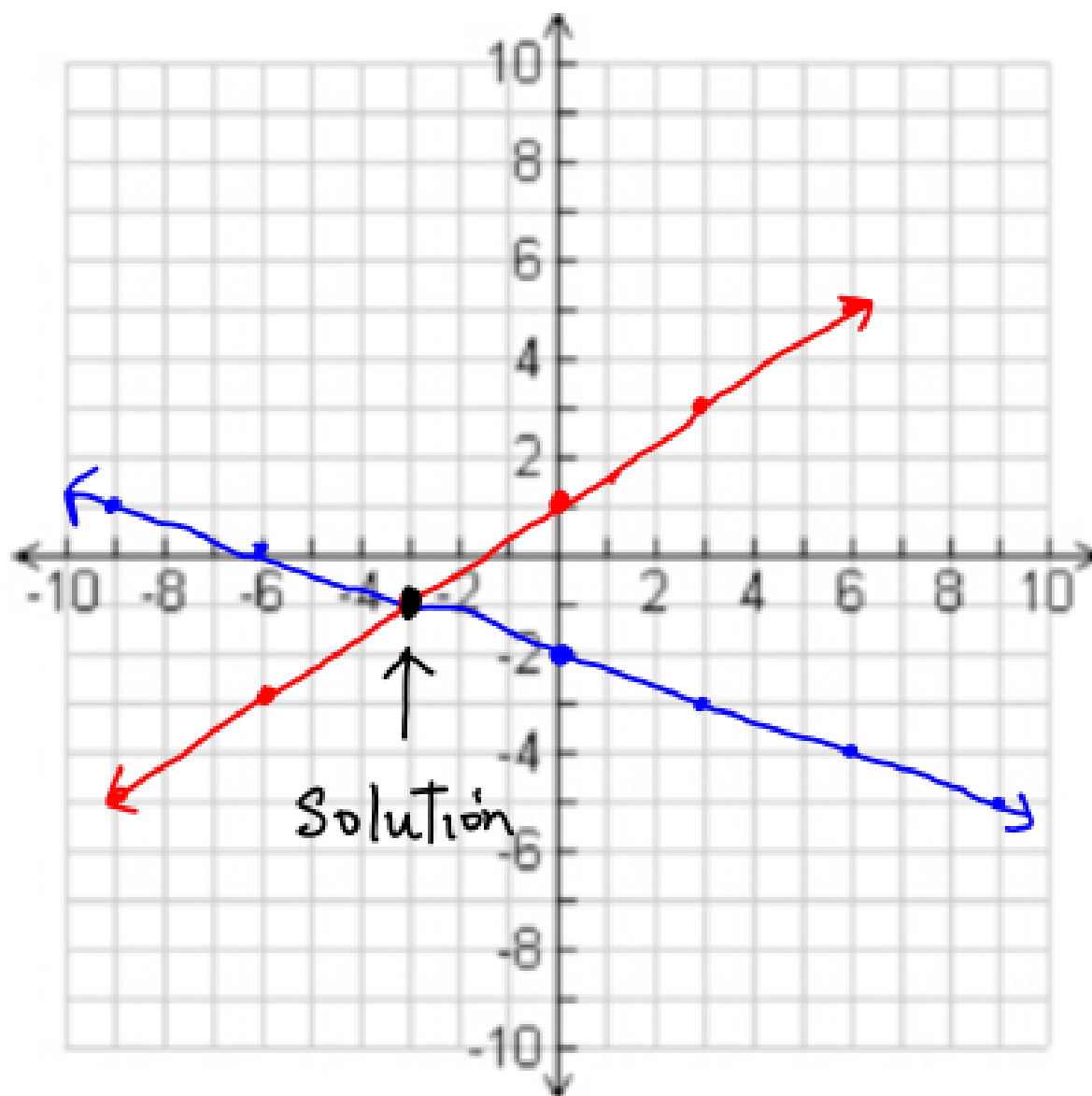**So, graph both lines and find the point of interception.**

a)
$$\begin{cases} y = -\dfrac{1}{3}x - 2 \quad \text{①} \\ y = \dfrac{2}{3}x + 1 \quad \text{②} \end{cases}$$

Solution :

$$\boxed{(x, y) = (-3, -1)}$$

Answer must be in
this Format !!

↑
Solution

b) $\begin{cases} 3x - 2y = 4 \\ x - 2y = -4 \end{cases}$

Need to put both equation in y = mx + b form first.
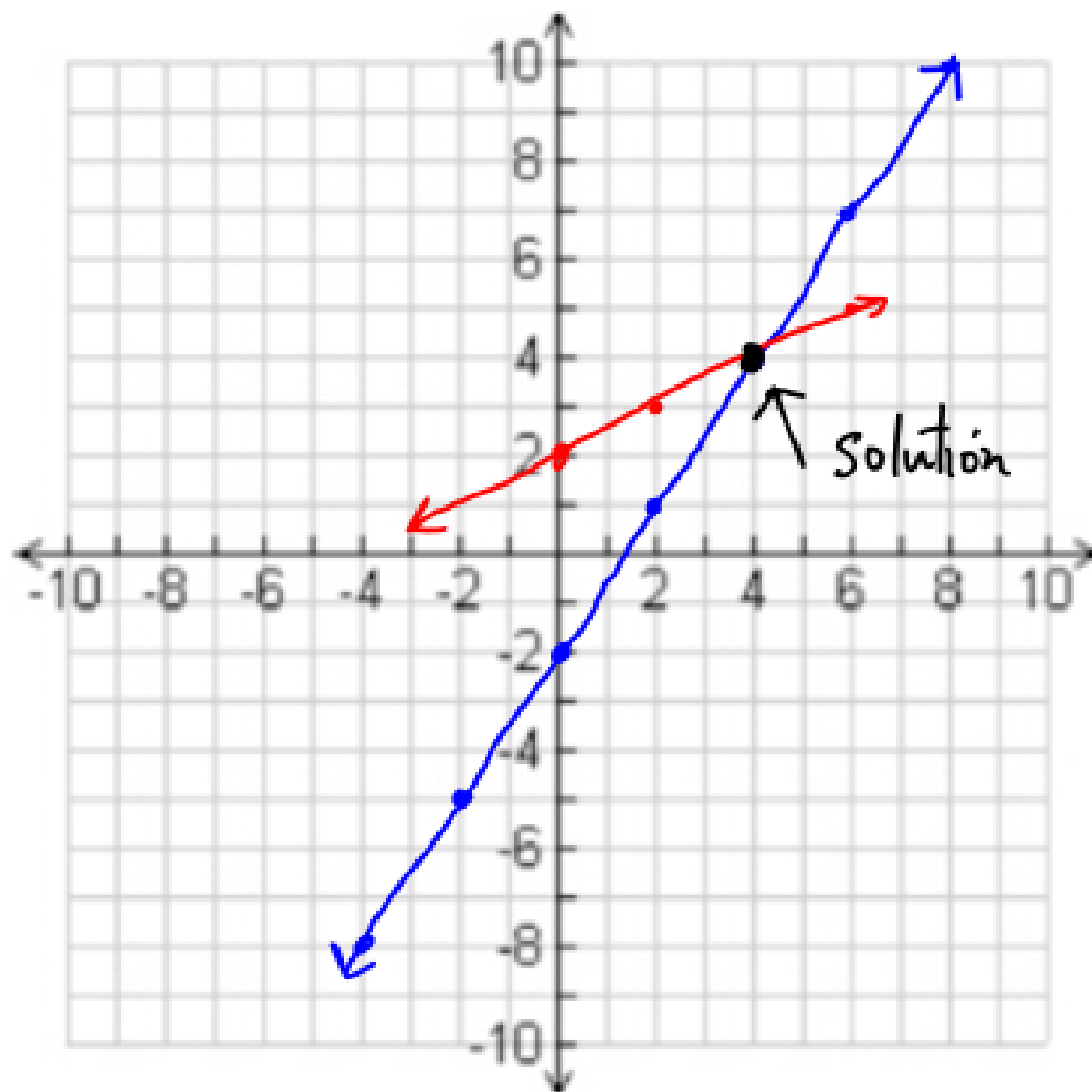
1) $3x - 2y = 4$

$-2y = -3x + 4$

$y = \frac{3}{2}x - 2$

2) $x - 2y = -4$

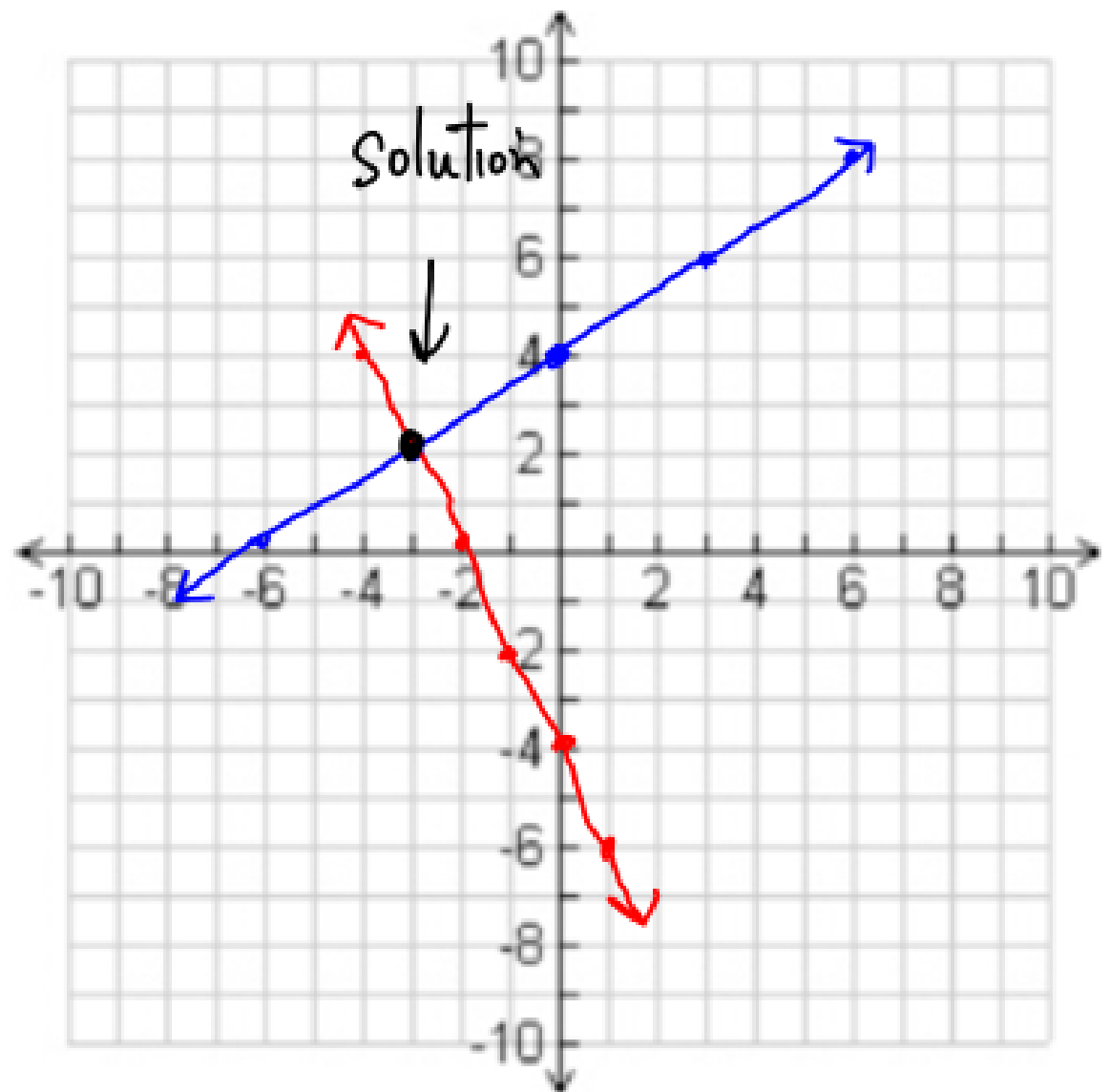$-2y = -1x - 4$

$y = \frac{1}{2}x + 2$

$(x, y) = (4, 4)$

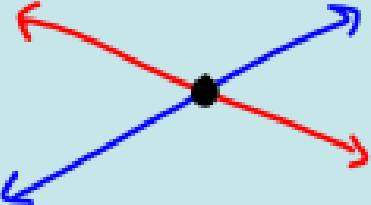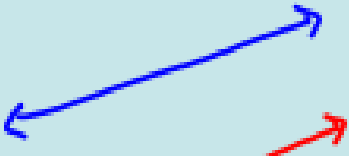$$Try) \begin{cases} 2x - 3y = -12 \\ 2x + y = -4 \end{cases}$$
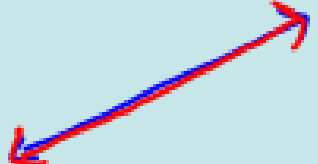
1) $-3y = -2x - 12$

$y = \frac{2}{3}x + 4$

2) $y = -2x - 4$

$(x, y) = (-3, 2)$

Solution

Linear Systems Having No Solution, One Solution or Infinite Solution

Intersecting Lines	Parallel Lines	Same Line
$(x,y) = (\text{point})$	Same slope !! $(x,y) = \{\emptyset\}$	Same Equation !! Same m & b $(x,y) = \{\text{pick one of the equation}\}$
One Solution Name: (Consistent Independent)	**No Solution** Name: (Inconsistent)	**Infinite Solution** Name: (Consistent Dependent)

Ex2) Solve the system: Same Slope : Parallel !!

a) $\begin{cases} 3x - 2y = 7 \\ 6x - 4y = 9 \end{cases}$

$(x,y) = \{\emptyset\}$

Inconsistent System.

1) $-2y = -3x + 7$ ——— m

$y = \boxed{\frac{3}{2}}x - \frac{7}{2}$

2) $-4y = -6x + 9$

$y = \frac{6}{4}x - \frac{9}{4}$ ——m

$y = \boxed{\frac{3}{2}}x - \frac{9}{4}$

No Need to Graph !!
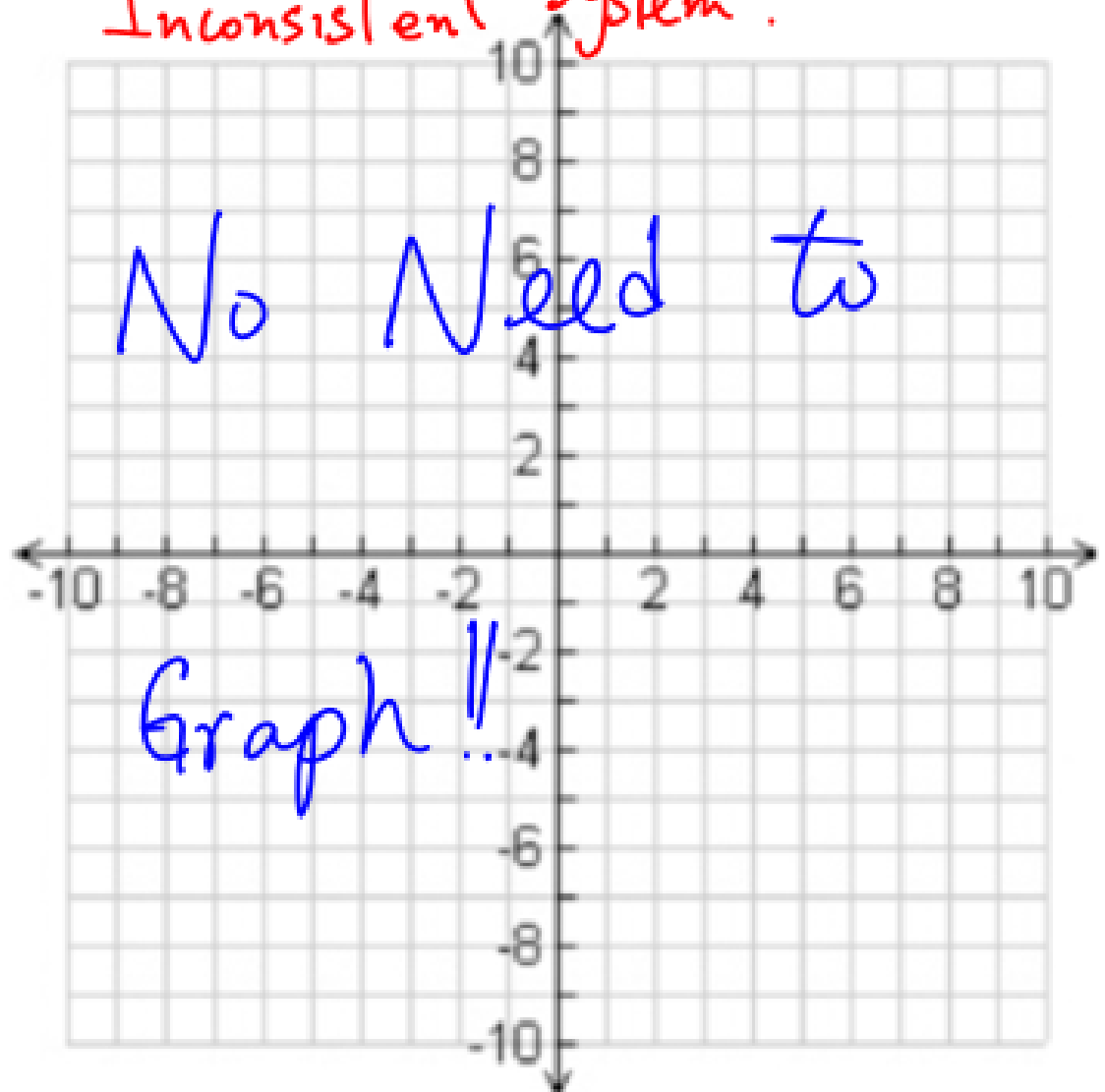
b) $\begin{cases} 2x - y = 7 \\ 6x - 3y = 21 \end{cases}$

Same Line !!

$(x,y) = \{2x - y = 7\}$

Consistent Dependent System.

1) $-y = -2x + 7$

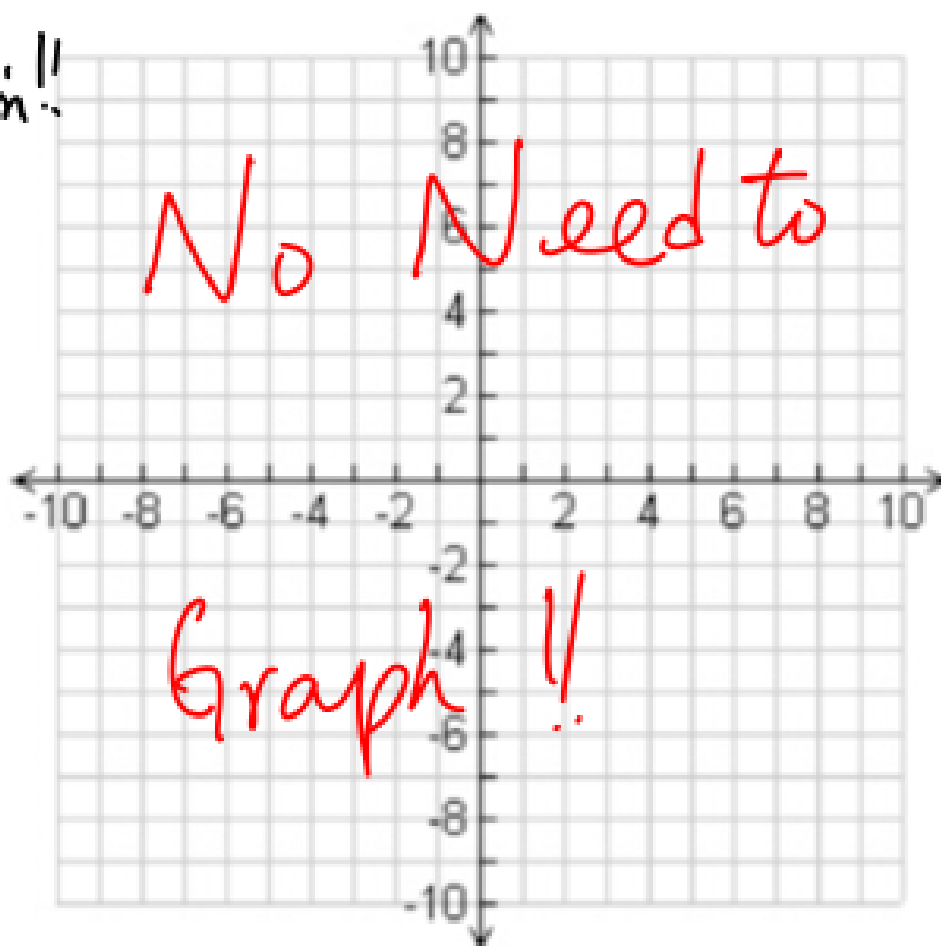$\boxed{y = 2x - 7}$

Same Equation !!

2) $-3y = -6x + 21$

$\boxed{y = 2x - 7}$

No Need to

Graph !!

5.2 Solving System of Equations by Substitution

Substitution Method: To put 1 equation into the other equation and solve for x or y.

Ex3) $\begin{cases} 3x + 2y = 14 \\ y = 2x \end{cases}$

Step:

1) In the 2nd equation, y = 2x, so put 2x into the 1st equation in place of y. Then solve for x.
2) Once you get x, put x into one of the 2 equation to find y.

$3x + 2(2x) = 14$

$3x + 4x = 14$

$7x = 14$

$x = 2$

$y = 2(2)$

$= 4$

$(x, y) = (2, 4)$

Ex4) $\begin{cases} 2x + 4y = -10 \\ x - y = 4 \end{cases}$

↑ easier to solve

Step:
1) You need to get x or y alone (whichever is easier.) So, $x = \boxed{y + 4}$
2) Now put $y + 4$ in for x in the first equation, then solve for y.
3) Once you get y, then put y into the easier equation to find x.

$2(y + 4) + 4y = -10$

$2y + 8 + 4y = -10$

$6y + 8 = -10$

$6y = -18$

$y = -3$

$x - (-3) = 4$

$x + 3 = 4$

$x = 1$

$\boxed{(x, y) = (1, -3)}$

Ex5) $\begin{cases} 6x - 2\boxed{y} = -4 \\ -3x + y = 2 \quad \longrightarrow \quad y = \boxed{3x + 2} \end{cases}$

$\uparrow$
easier
to solve

$6x - 2(3x + 2) = -4$

$\cancel{6x} - \cancel{6x} - 4 = -4$

$-4 = -4 \quad$ True !!

$(x, y) = \{ 6x - 2y = -4 \} \quad ; \quad$ Consistent Dependent
System

Ex6) $\begin{cases} 8x + 2y = 13 \\ 4x + y = 11 \end{cases} \rightarrow y = \boxed{-4x + 11}$

$\uparrow$

easier
to solve

$8x + 2(-4x + 11) = 13$

$8x - 8x + 22 = 13$

$22 = 13$ False

$(x, y) = \{\phi\}$; Inconsistent System

Try) $\begin{cases} \cancel{x + y = 1} & \begin{cases} 2x - y = 14 \\ \cancel{2x - y = 2} & \phantom{\begin{cases}\end{cases}} \end{cases} \end{cases}$

$\begin{cases} 2x - y = 14 \\ \boxed{x} + 4y = -2 \quad \longrightarrow \quad x = \boxed{-4y - 2} \end{cases}$

$\uparrow$
Solve

$2(-4y - 2) - y = 14 \qquad x + 4(-2) = -2$

$-8y - 4 - y = 14 \qquad x - 8 = -2$

$-9y - 4 = 14 \qquad\qquad x = 6$

$-9y = 18$

$\boxed{(x, y) = (6, -2)}$

$y = -2$