7.2 Factor the Polynomial by Using GCF & Grouping

From Chapter 6: Multiplying Polynomials

a) $x(2x + 7) = 2x^2 + 7x$

Answer / Factor Form Given

b) $5xy(x - 2y) = 5x^2y - 10xy^2$

c) $-2x^2(x^2 + 5x - 3) = -2x^4 - 10x^3 + 6x^2$

*Today: Factoring is the **OPPOSITE** of distributing

Polynomials that CAN NOT be factored are called PRIME.

Ex1) Find the GCF (Greatest Common Factor):

*Pick the **SMALLEST** exponent of EACH variable.*

a) $40a^2b^2$ and $30ab$

GCF: $10a^1b^1$

b) $12a^2b$ and $90a^2b^3c$

12: 1, 2, 3, 4, 6, 12

90: 1, 2, 3, 5, 6, 9, 10, 15...

GCF: $6a^2b^1$

c) $-7(x-9)^8$ and $14(x-9)^2$ ← smaller

same same

GCF: $-7(x-9)^2$

Ex2) Factor out the GCF.

1) Find **GCF**
2) Pick SMALLEST Exponent
3) **DIVIDE** in the numbers & **SUBTRACT** the exponents

a) $8x + 36y$ GCF = 4

$\frac{8x}{4} \quad \frac{36y}{4}$

$= 4(2x + 9y)$

Answer!!

Check: multiply out: $8x + 36y$ ✱

b) $20x^2 - 24xy^2$ GCF = 4x

$\frac{20x^2}{4x} \quad \frac{24xy^2}{4x}$

$= 4x(5x - 6y^2)$

Check: $20x^2 - 24xy^2$ ✱

c) $28a^2b^2c^2 + 21a^2bc^2 - 14abc$

$\frac{28a^2b^2c^2}{7abc} \quad \frac{21a^2bc^2}{7abc} \quad \frac{14abc}{7abc}$

GCF = 7abc

$= 7abc(4abc + 3ac - 2)$

d) $3c^2d - 6c^2d^2$ GCF = $3c^2d$

$\frac{3c^2d}{3c^2d} \quad \frac{6c^2d^2}{3c^2d}$

$= 3c^2d(1 - 2d)$

e) $ab + a$ GCF = a

$\frac{ab}{a} \quad \frac{a}{a}$

$= a(b + 1)$

$a^{1-1} = a^0 = 1$

Try This: Factor out the GCF.

a) $12x^2 + 16x$ GCF = $4x$

$= 4x(3x + 4)$

b) $21x^2y - 24xy^2$ GCF = $3xy$

$= 3xy(7x - 8y)$

c) $27a^2b^3c^4 + 21a^2b^2c^2 - 15a^3b^4c^3$ GCF = $3a^2b^2c^2$

$= 3a^2b^2c^2(9bc^2 + 7 - 5ab^2c)$

d) $4c^4d^2 - 6c^2d^2$ GCF = $2c^2d^2$

$= 2c^2d^2(2c^2 - 3)$

e) $a^3b + a^2b^2$ GCF = a^2b

$= a^2b(a + b)$

Ex3) Factoring by Grouping for 4 terms.

$a)\left(x^3+7x^2\right)+\left(2x+14\right)$

GCF GCF

Step 1: Find the GCF of the 1st 2 terms.
Find the GCF of the last 2 terms.

$= x^2(x+7)+2(x+7)$

= same = same

Step 2: Take what is outside the ( ) & put it as your 1st binomial

$=\left(x^2+2\right)(x+7)$

Answer !!

Step 3: Take what is inside the (**must be the same**) & put it as your 2nd binomial

Step 4: Use FOIL to check your answer.

Check: $(x^2+2)(x+7)$

$= x^3+7x^2+2x+14$ ✱

b) $(3x^3+15x^2)(-2x-10)$

GCF GCF

$= 3x^2(x+5) - 2(x+5)$

same same

$= \boxed{(3x^2-2)(x+5)}$

d) $(x^4+4x^3)(+2x+8)$

$= x^3(x+4) + 2(x+4)$

$= \boxed{(x^3+2)(x+4)}$

c) $(x^3+2x^2)(-5x-10)$

$= x^2(x+2) - 5(x+2)$

$= \boxed{(x^2-5)(x+2)}$

or $(x+2)(x^2-5)$

e) $(6y^2-3y)(+2yz-z)$

$= 3y(2y-1) + z(2y-1)$

$= \boxed{(3y+z)(2y-1)}$

Try This: Factor by Grouping.

a) $(x^2y + 3x^2)(-7y^2 - 21y)$

$= x^2(y+3) - 7y(y+3)$

$= (x^2 - 7y)(y+3)$

b) $(10x^3 - 25x^2)(+4x - 10)$

$= 5x^2(2x-5) + 2(2x-5)$

$= (5x^2 + 2)(2x-5)$

c) $(2x^2y + 6xy)(-x - 3)$

$= 2xy(x+3) - 1(x+3)$

$= (2xy - 1)(x+3)$