8.1 Identifying Quadratic Functions

Standard Form: $y = ax^2 + bx + c$ (a, b, c are real numbers)

Highest Power for x

$a \neq 0$

ex) $y = 2x^2 + 4x - 5$

$a = 2 \quad b = 4 \quad c = -5$

ex) $y = -3x^2 - 1$

$a = -3$

$b = 0$

$c = -1$

Tell whether the equation is a quadratic function. Explain.

1) $y = x^2 + 2x - 3$

Yes, because the highest power is 2.

2) $5x + y = -2$

No, because the highest power is 1, Linear!

3) $y + 5x^2 = -8$

Yes, because the highest power is 2.

Tell whether each function is quadratic. Explain.

Step 1: Find the first difference in x-value. **(Must be the same!)** ✓
How: Take the 2nd x-value minus the 1st one.

Step 2: Find the second difference in y-value. **(Must be the same!)**
How: First find the 1st difference, then find the difference again.

Note: The 1st difference of y can NOT be the same, but the 2nd difference of y must be the SAME !!

x	y
−5	−191
−3	−59
−1	1
1	−11
3	−95

Same ↓

$-3-(-5)=2$

$-1-(-3)=2$

$1-(-1)=2$

$3-1=2$

Different

$-59-(-191)=132$

$1-(-59)=60$

$-11-1=-12$

$-95-(-11)=-84$

Same ↓

$60-132=-72$

$-12-60=-72$

$-84-(-12)=-72$

Yes, because the 1st difference of x and 2nd difference of y are the same!

Tell whether each function is quadratic. Explain.

1)

x	y
−4	43
−1	16
2	7
5	16
8	43

same ↓ (x) — different ↓ (y) — same ↓ (2nd difference of y)

$-1-(-4)=3$, $2-(-1)=3$, $5-2=3$, $8-5=3$

$16-43=-27$, $7-16=-9$, $16-7=9$, $43-16=27$

$-9-(-27)=18$, $9-(-9)=18$, $27-9=18$

Yes, because the 1st difference of x and 2nd difference of y are the SAME!

2)

x	y
−2	12
−1	4
0	0
1	6
2	28

1, 1, 1, 1

−8, −4, 6, 22

4, 10, 16

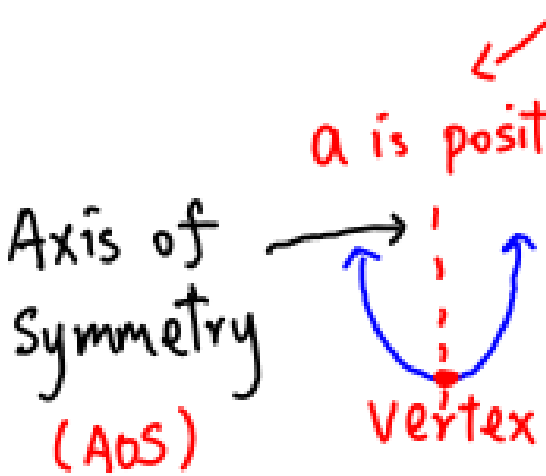
No, because the 2nd difference of y is not the same.

Graph of Quadratic Functions: Parabola

(u-shape) ∪ or ∩

Standard Form: $y = ax^2 + bx + c$

a is positive



minimum value = lowest y-value

mini value = 5

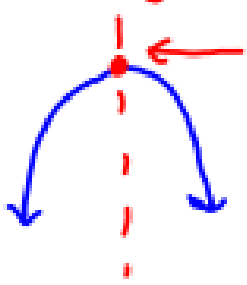
AOS: x = x-value of the vertex

x = 3

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq \text{lowest } y\}$: $\{y \mid y \geq 5\}$

a is negative



, maximum value = highest y-value

max value = 10

AOS: x = 4

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \leq \text{highest } y\}$

$\{y \mid y \leq 10\}$

The bigger the a is, the narrow the ∪ is.
The smaller a is the wide ∪ is.
*Ignore "−"

Order each group of Quadratic Functions from WIDEST to NARROWEST

1) $y = 3x^2;\ y = \frac{1}{2}x^2;\ y = 1x^2$

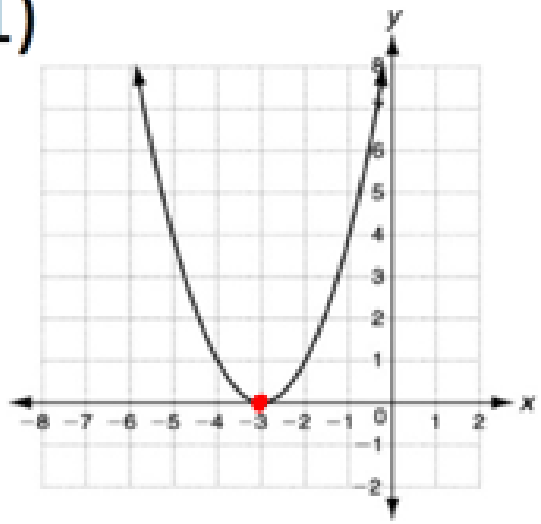
$y = \frac{1}{2}x^2$; $y = x^2$; $y = 3x^2$

2) $f(x) = -\frac{1}{2}x^2;\ f(x) = 4x^2;\ f(x) = -\frac{1}{4}x^2$

$f(x) = -\frac{1}{4}x^2$; $f(x) = -\frac{1}{2}x^2$; $f(x) = 4x^2$

Find the coordinates of the vertex. Determine if the function has a maximum or minimum and find its value. Find domain, and range (set ~~and interval notation~~), AOS.

1)



Vertex: $(-3, 0)$

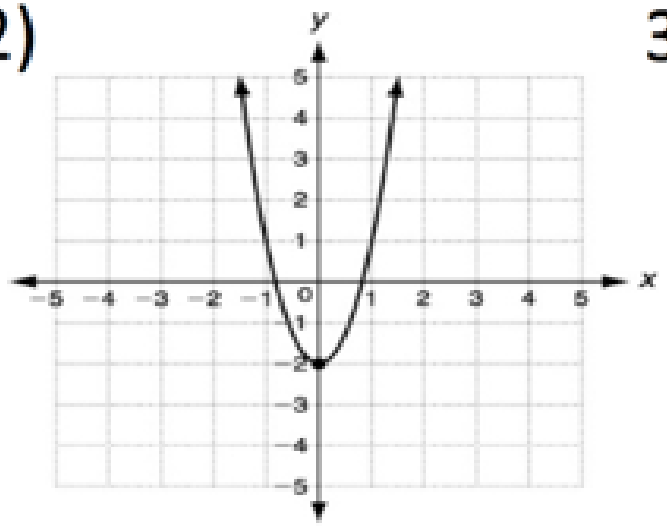
Min value $= 0$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq 0\}$

AOS: $x = -3$

2)



Vertex: $(0, -2)$

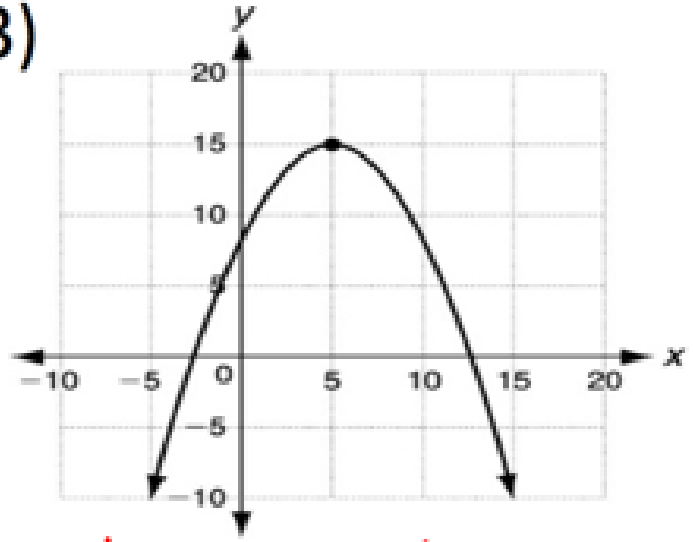
Min. value $= -2$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq -2\}$

AOS: $x = 0$

3)



Vertex: $(5, 15)$

Max. value $= 15$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \leq 15\}$

AOS: $x = 5$

Graph each quadratic function. Identify the vertex, minimum or maximum value. Find the Domain & Range (set and interval notation)

1) $y = x^2$ $a = 1$ $b = 0$

Vertex: $x = \frac{-b}{2a} = \frac{-0}{2(1)} = 0$

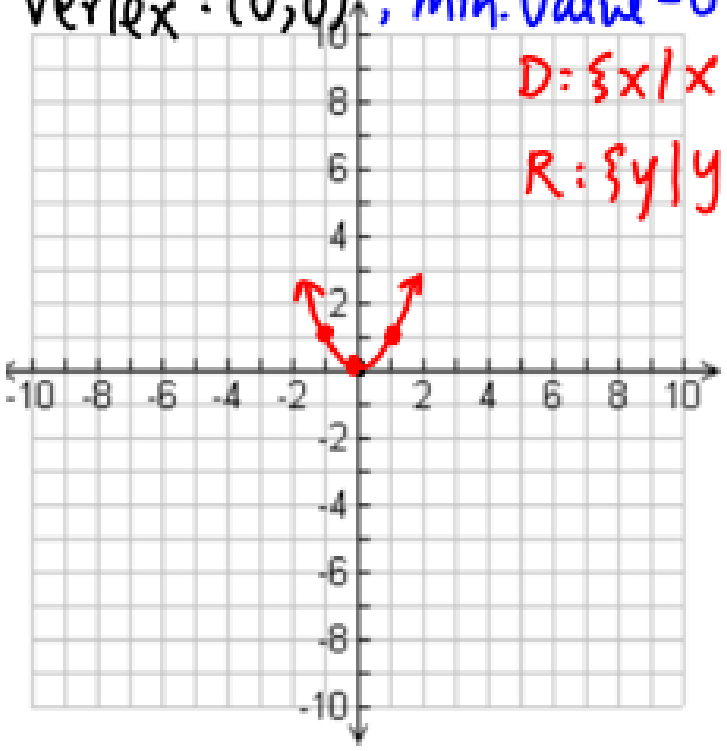
y = plug in the x into the equ.

$y = 0^2 = 0$

Vertex: $(0,0)$; min. value $= 0$; AOS: $x = 0$

$D: \{x \mid x \in R\}$

$R: \{y \mid y \geq 0\}$



x	y
-1	$(-1)^2 = 1$
0	0
1	$1^2 = 1$

2) $f(x) = 2x^2 - 3$ $a = 2$ $b = 0$

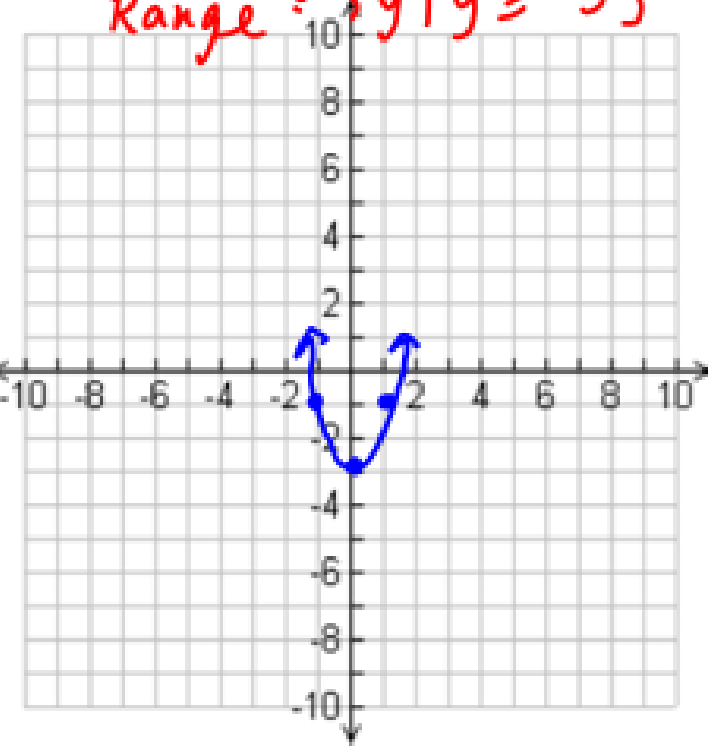
Vertex: $x = \frac{-b}{2a} = 0$

$y = 2(0)^2 - 3 = -3$

Vertex: $(0, -3)$; min value $= -3$

AOS: $x = 0$; Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq -3\}$



x	y
-1	$2(-1)^2 - 3 = -1$
0	-3
1	$2(1)^2 - 3 = -1$

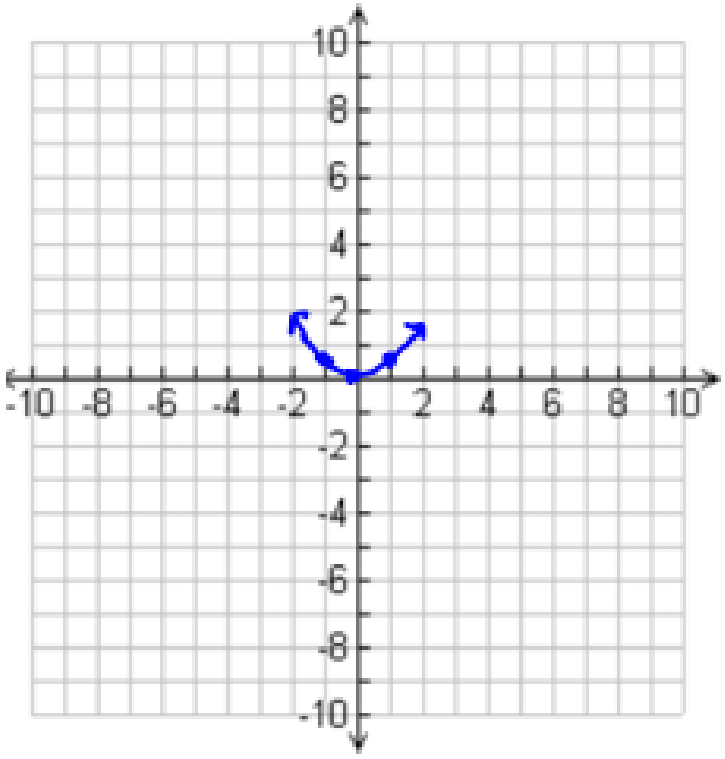
3) $y = \frac{1}{2}x^2$ $a = \frac{1}{2}$, $b = 0$

$x = 0, y = 0 \rightarrow$ vertex : $(0, 0)$

min. value $= 0$; AOS : $x = 0$

Domain : $\{x \mid x \in R\}$

Range : $\{y \mid y \geq 0\}$



x	y
-1	$\frac{1}{2}$
0	0
1	$\frac{1}{2}$

$a = -2$

$b = 0$

4) $f(x) = -2x^2 + 2$

Vertex : $x = \frac{-b}{2a} = 0$; $y = -2(0)^2 + 2 = 2$

Vertex : $(0, 2)$

max. value $= 2$; AOS : $x = 0$

Domain : $\{x \mid x \in R\}$

Range : $\{y \mid y \leq 2\}$

x	y
-1	$-2(-1)^2 + 2 = 0$
0	2
1	0