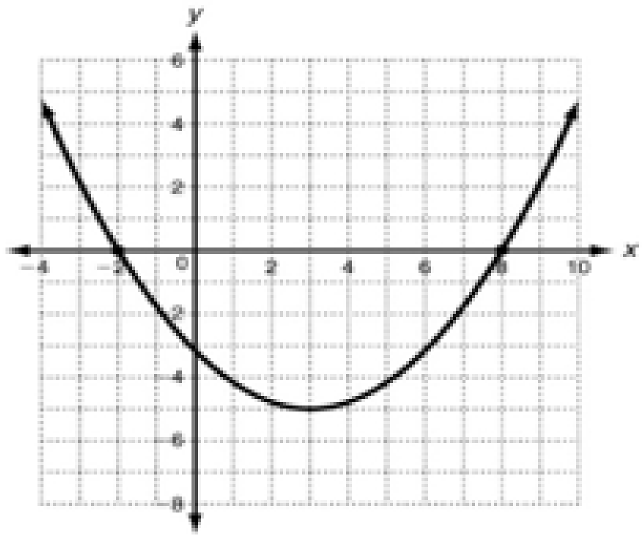
8.2 Characteristics of Quadratic Functions

Find the vertex, maximum or minimum value, AOS. Then state domain, and range.



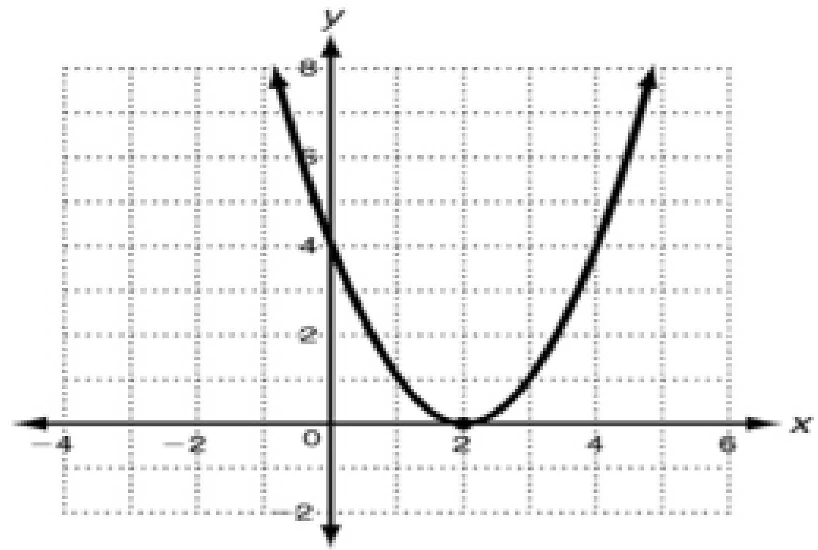
Vertex: (3, -5)

AOS: x = 3

Max. or Min Value = -5

Domain: {x | x ∈ R}

Range: {y | y ≥ -5}



Vertex: (2, 0)

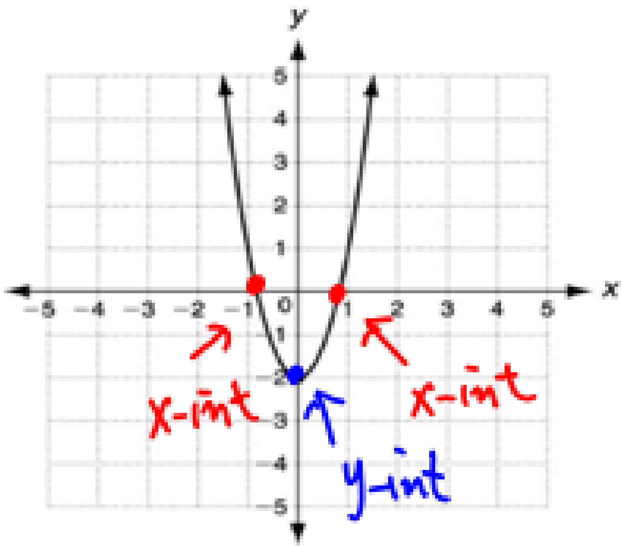
AOS: x = 2

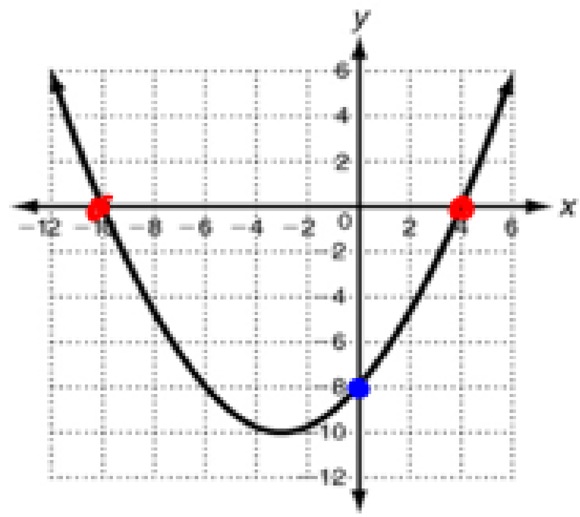
Max. or Min Value = 0

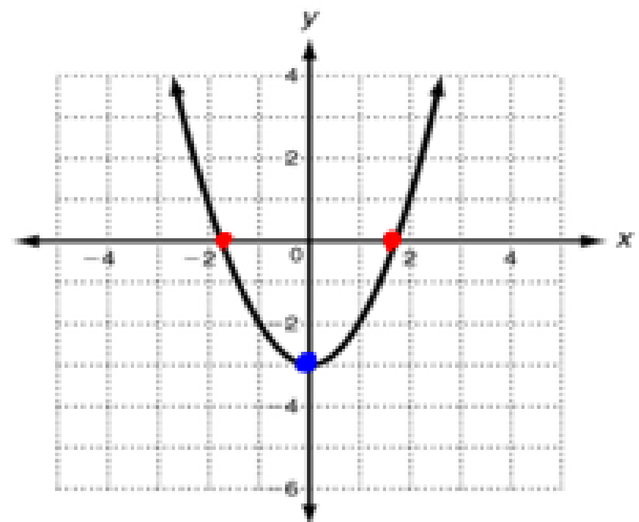
Domain: {x | x ∈ R}

Range: {y | y ≥ 0}

Find the x- and y-intercept.







x-int: (-1, 0) ; (1, 0)

y-int: (0, -2)

x-int: (-10,0), (4,0)

y-int: (0, -8)

x-int: (-1.8, 0),
(1.8, 0)

y-int: (0, -3)

x-int: points cross the x-axis ⟹ also called the zero !!

y-int: point cross the y-int

Find the vertex, axis of symmetry & y intercept of the function's graph. Then state the domain & range.

1) $y=-x^2+8x-1$ ① $a=-1$, $b=8$, $c=-1$

② vertex: $x=\frac{-b}{2a}=\frac{-8}{2(-1)}=\frac{-8}{-2}=4$

y = plug x=4 (from the last step) into the given equation.

$y=-(4)^2+8(4)-1=-16+32-1=15$

vertex: $(4,15)$

AOS: $x=4$

y-int: $(0, c)$ Always!! $=(0,-1)$

max

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \le 15\}$

2) $y = 3x^2 + 6x + 5$ $a = 3$, $b = 6$, $c = 5$

vertex: $x = \frac{-b}{2a} = \frac{-6}{2(3)} = \frac{-6}{6} = -1$

$y = 3(-1)^2 + 6(-1) + 5 = 3 - 6 + 5 = 2$

Vertex: $(-1, 2)$

AOS: $x = -1$

y-int: $(0, 5)$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq 2\}$

3) $y = \frac{1}{2}x^2 - 4x + 11$ $a = \frac{1}{2}$, $b = -4$, $c = 11$

Vertex : $x = \frac{-b}{2a} = \frac{4}{2(\frac{1}{2})} = 4$

$y = \frac{1}{2}(4)^2 - 4(4) + 11 = \frac{1}{2}(16) - 16 + 11 = 8 - 16 + 11 = 3$

Vertex : $(4, 3)$

AOS : $x = 4$

y-int : $(0, 11)$

Domain : $\{x \mid x \in R\}$

Range : $\{y \mid y \geq 3\}$

4) $y = 2x^2 - 12x + 10$ $a = 2,\ b = -12,\ c = 10$

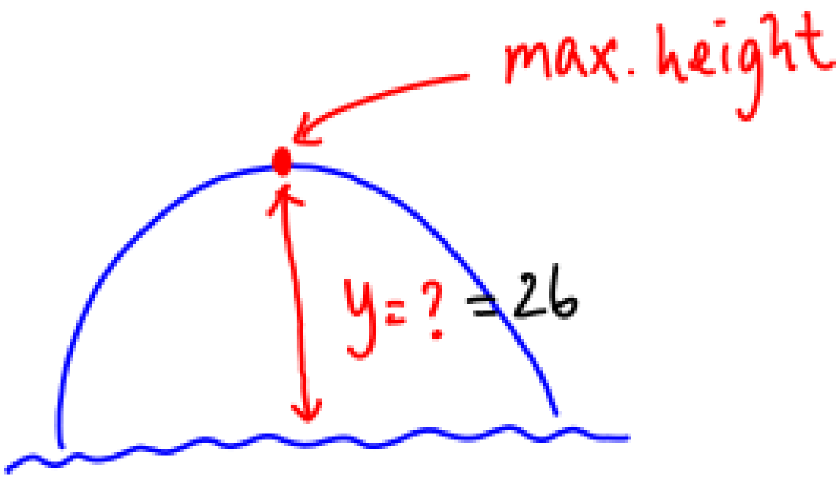
$x = \frac{-b}{2a} = \frac{12}{2(2)} = \frac{12}{4} = 3$

$y = 2(3)^2 - 12(3) + 10 = 18 - 36 + 10 = -8$

$$\boxed{\begin{array}{l} \text{Vertex}: (3, -8) \\ \text{AOS}: x = 3 \\ \text{y-int}: (0, 10) \\ \text{Domain}: \{x \mid x \in \mathbb{R}\} \\ \text{Range}: \{y \mid y \geq -8\} \end{array}}$$

The graph of $\boxed{f(x) = -0.007x^2 + 0.84x + 0.8}$ can be used to model the height in feet of an arch support for a bridge, where *x* represents the horizontal distance in feet from where the arch support enters the water. Can a sailboat that is 24 feet tall pass under the bridge? Explain.

$a = -0.007;\ b = 0.84$



$$x = \frac{-b}{2a} = \frac{-0.84}{2(-0.007)} = \frac{-0.84}{-0.014} = 60$$

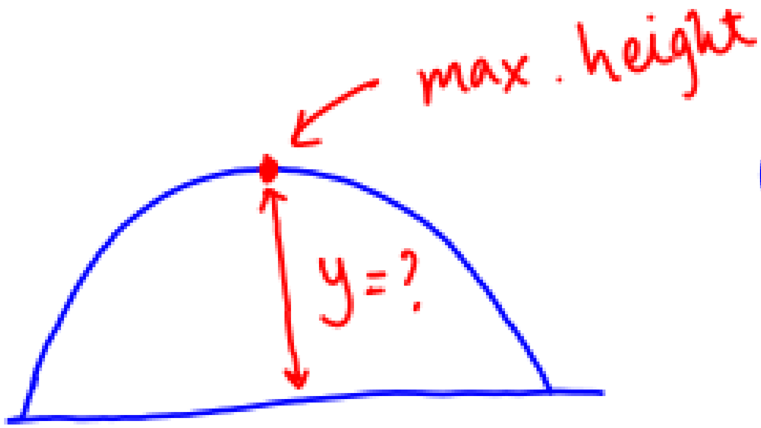
$$y = -0.007(60)^2 + 0.84(60) + 0.8$$

$$= -0.007(3600) + 50.4 + 0.8$$

$$= -25.2 + 50.4 + 0.8 = 26$$

Yes, because the max. height is 26 feet.

The height of a small rise in a roller coaster track is modeled by $f(x) = -0.07x^2 + 0.42x + 6.37$, where x is the distance in feet from a supported pole at ground level. Find the greatest height of the rise.



max. height = y

$a = -0.07$, $b = 0.42$

$$x = \frac{-b}{2a} = \frac{-0.42}{2(-0.07)} = \frac{-0.42}{-0.14} = 3$$

$$y = -0.07(3)^2 + 0.42(3) + 6.37$$

$$= -0.07(9) + 1.26 + 6.37$$

$$= -0.63 + 1.26 + 6.37$$

$$= 7$$

The greatest height of the rise is 7 feet