

8.8 Solving Quadratic Equation 2

Solving by Completing the Square: $x^2 + bx = c$

Ex1) $x^2 + 8x = 48$ $b=8$

$x^2 + 8x + 16 = 48 + 16$ $\frac{b}{2} = 4$

$(x + 4)^2 = 64$

(Note: In the original image, 'x' and '4' are underlined with a red wavy line, and '4' is written below it with a red arrow and the fraction $\frac{b}{2}$.)

$x + 4 = \pm 8$

$x + 4 = 8$; $x + 4 = -8$
 -4 -4 -4 -4

$x = 4$; $x = -12$

- 1) Find $\left(\frac{b}{2}\right)^2$ and add to both sides.
+16 to both sides
- 2) Write the left side as $(x \bigcirc \frac{b}{2})^2$:
- 3) Combine the right side.
- 4) Square both sides and add \pm on the number.
- 5) Set up two equations and
- 6) Solve for x

$$\text{Ex2) } x^2 - 10x = -21$$

$$\frac{b}{2} = \frac{-10}{2} = -5 ; (-5)^2 = 25$$

$$x^2 - 10x + 25 = -21 + 25$$

$$\sqrt{(x-5)^2} = \sqrt{4}$$

$$x-5 = \pm 2$$

$$\begin{array}{r} x-5 = 2 \\ +5 \quad +5 \\ \hline \end{array} ; \begin{array}{r} x-5 = -2 \\ +5 \quad +5 \\ \hline \end{array}$$

$$\boxed{x=7} ; \boxed{x=3}$$

- 1) Find $\left(\frac{b}{2}\right)^2$ and add to both sides.
+25 to both sides
- 2) Write the left side as $(x \bigcirc \frac{b}{2})^2$:
- 3) Combine the right side.
- 4) Square both sides and add \pm on the number.
- 5) Set up two equations and
- 6) Solve for x

$$\text{Ex3) } x^2 + 4x - 15 = 0$$

+15 +15

$$x^2 + 4x + 4 = 15 + 4$$

$$\sqrt{(x+2)^2} = \sqrt{19}$$

$$x + 2 = \pm \sqrt{19}$$

-2 -2

$$x = -2 \pm \sqrt{19}$$

$$\frac{b}{2} = \frac{4}{2} = 2 ; 2^2 = 4$$

- 1) Move c to the other side.
- 2) Find $\left(\frac{b}{2}\right)^2$ and add to both sides.
+4 to both sides
- 3) Write the left side as $(x \bigcirc \frac{b}{2})^2$:
- 4) Combine the right side.
- 5) Square both sides and add \pm on the number.
- ~~6) Set up two equations and~~
- 7) Solve for x

$$\text{Ex4) } x^2 + 12x - 13 = 0$$

$$x^2 + 12x + 36 = 13 + 36$$

$$\frac{12}{2} = 6 \rightarrow 6^2 = \underline{\underline{36}}$$

Add

$$\sqrt{(x + 6)^2} = \sqrt{49}$$

$$x + 6 = \pm 7$$

$$x + 6 = 7 \quad ; \quad x + 6 = -7$$

$$\begin{array}{r} -6 \quad -6 \\ \hline \end{array}$$

$$x = 1$$

$$\begin{array}{r} -6 \quad -6 \\ \hline \end{array}$$

$$x = -13$$

- 1) Move c to the other side.
- 2) Find $\left(\frac{b}{2}\right)^2$ and add to both sides.
- 3) Write the left side as $(x \bigcirc \frac{b}{2})^2$:
- 4) Combine the right side.
- 5) Square both sides and add \pm on the number.
- 6) Set up two equations and
- 7) Solve for x

$$\text{Try 1) } x^2 - 20x + 88 = 0$$

$$x^2 - 20x + 100 = -88 + 100$$

$$\sqrt{(x-10)^2} = \sqrt{12}$$

$$\begin{array}{ccc} x-10 & = & \pm 2\sqrt{3} \\ +10 & & +10 \end{array}$$

$$x = 10 \pm 2\sqrt{3}$$

~~$$\text{Try 2) } x^2 - 20x + 88 = 0$$~~

$$\frac{b}{2} = \frac{-20}{2} = -10$$

$$(-10)^2 = 100$$

Add

$$\begin{array}{c} \sqrt{12} \\ \wedge \\ \sqrt{4} \sqrt{3} \end{array}$$

Solving by Using Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(Must = 0 before you find the a, b, and c.)

Ex5) $x^2 - 2x - 15 = 0$ $a = 1$
 $x = \frac{2 \pm \sqrt{4 - 4(1)(-15)}}{2(1)}$ $b = -2$
 $c = -15$

$= \frac{2 \pm \sqrt{4 + 60}}{2}$ ^{1st} ^{2nd}

$= \frac{2 \pm \sqrt{64}}{2} = \frac{2 \pm 8}{2}$
 $\nearrow x = \frac{2+8}{2} = \frac{10}{2} = \boxed{5}$
 $\searrow x = \frac{2-8}{2} = \frac{-6}{2} = \boxed{-3}$

Ex6) $x^2 + 6x + 8 = 0$ $a = 1$
 $x = \frac{-6 \pm \sqrt{36 - 4(1)(8)}}{2(1)}$ $b = 6$
 $c = 8$

$= \frac{-6 \pm \sqrt{36 - 32}}{2}$

$= \frac{-6 \pm \sqrt{4}}{2}$
 $= \frac{-6 \pm 2}{2}$
 $\nearrow x = \frac{-6+2}{2} = \boxed{-2}$
 $\searrow x = \frac{-6-2}{2} = \boxed{-4}$

(Must = 0 before you find the a, b, and c.)

$$\text{Ex7) } 2x^2 + 7x = 15$$

$-15 \quad -15$

$$2x^2 + 7x - 15 = 0$$

$$a = 2$$

$$b = 7$$

$$c = -15$$

$$x = \frac{-7 \pm \sqrt{49 - 4(2)(-15)}}{2(2)}$$

$$= \frac{-7 \pm \sqrt{49 + 120}}{4}$$

$$= \frac{-7 \pm \sqrt{169}}{4} = \frac{-7 \pm 13}{4}$$

$$x = \frac{-7 + 13}{4} = \frac{6}{4} = \frac{3}{2}$$

$$x = \frac{-7 - 13}{4} = \frac{-20}{4} = -5$$

$$\text{Ex8) } 2x^2 - 7x = 3$$

$$2x^2 - 7x - 3 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 4(2)(-3)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{49 + 24}}{4} = \frac{7 \pm \sqrt{73}}{4}$$

$$\text{Try1) } x^2 - 5x + 12 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 4(1)(12)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25 - 48}}{2}$$

$$= \frac{5 \pm \sqrt{-23}}{2}$$

Not a Real Root !!

$$\text{Try2) } 2x^2 + 2 = 9x$$

$$2x^2 - 9x + 2 = 0$$

$$x = \frac{9 \pm \sqrt{81 - 4(2)(2)}}{2(2)}$$

$$= \frac{9 \pm \sqrt{81 - 16}}{4}$$

$$= \frac{9 \pm \sqrt{65}}{4}$$