9.2-9.3 Exponential Functions

Exponential Model:

variable Exponent

Ex1) If a rubber ball is dropped from a height of 10 feet, the function f(x) = 20 $(0.6)^{\boxed{x}}$ gives the height in feet of each bounce, where x is the bounce number. What will be the height of the 5th bounce? (Round to the nearest tenth of a foot.)

$x=5$

Calculator:

$f(5) = 20(0.6)^5$

1) 0.6 $\boxed{x^y}$ or $\boxed{y^x}$ or $\boxed{\wedge}$ 5 ↵

0.07776

≈ 1.6

2) × 20 ↵ 1.5552

The height will be about 1.6 feet at the 5th bounce.

Ex2) A population of pigs is expected to increase at a rate of 4% each year. If the original population is 1000, the function $f(x) = 1000\,(1.04)^x$ gives the population in x years. What will be the population in 12 years?

$x = 12$

Calculator:

1) $1.04 \ \boxed{x^y} \ 12$

2) $\times 1000$

$f(12) = 1000\,(1.04)^{12}$

$\approx 1{,}601$

The population will be about 1601 pigs in 12 years.

Writing an Exponential Model:

Calculator

1) $1.04 \boxed{x^y} 9$

2) $\times 236{,}000$

Exponential **Growth**: $y = a\,(1 + r)^t$ ← time

y: Final amount

a: Initial amount

r: rate in decimal

ex) $4\% = 0.04$

Ex3) The population of a city is increasing at a rate 4% each year. In 2000 there were 236,000 people in the city. ① Write an exponential growth function to model this situation. Then find the population in 2009. ②

When you are writing a model: you need to leave the y & t as the variables.

$a = 236{,}000$
$r = 0.04$
} plug in a & r to the Equation

① Model: $y = 236{,}000(1 + 0.04)^t$

② $t = 9$: $y = 236{,}000\,(1 + 0.04)^9$

$\approx 335{,}902$

The population will be about 335,902 in 2009.

Exponential **Decay**: $y = a(1 - r)^t$

Ex4) The population of a city is decreasing at a rate of 6% ① each year. In 2000 there were 35,000 people in the city. Write an exponential decay function to model this situation. Then find the population in 2012. ②

Calculator:

1) $.94 \boxed{x^y} 12$

2) $\times 35{,}000$

① Model: $y = 35{,}000(1 - 0.06)^t$

② $t = 12$: $y = 35{,}000(1 - 0.06)^{12}$

$\approx 16{,}657$

$\boxed{\text{The population will be about 16657 in 2012.}}$

Try1) Annual sales at a company are $372,000 and increasing at a rate of 5% per year. Write the exponential model and find the sales after 8 years.

Model: $y = 372{,}000(1 + 0.05)^t$

$t = 8$: $y = 372\,000(1.05)^8$

$\approx 549{,}613.43$

$\boxed{\text{The sales will be about \$549,613.43 after 8 years.}}$

Try2) Monthly car sales for a certain type of car are $350,000 and are decreasing at a rate of 3% per month. Write the exponential model and find the sales after 6 months.

Model: $y = 350{,}000(1 - 0.03)^t$

$t = 6$: $y = 350{,}000(0.97)^6$

$\approx 291{,}540.20$

$\boxed{\text{The sales will be about \$291,540.20 after 6 months.}}$

Special Type of Exponential Growth Functions:

involves finding the compound interest. (Investment)

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A: Final Balance

P: original amount

r: rate in decimal

n: compounded method in a year.

t = time

1) monthly : $n=12$
2) quarterly : $n=4$
3) annually : $n=1$
4) semi-annually : $n=2$
5) weekly : $n=52$

Ex5) Write a compound interest function to $\boxed{\text{model}}$ \$15,000 invested at a rate of 3% compounded quarterly. Then find the balance after 8 years.

$n=4$

$P = 15{,}000$

$r = 0.03$

$n = 4$

Model: $A = 15{,}000\left(1+\frac{.03}{4}\right)^{4t}$

$t=8$: $A = 15{,}000\left(1+\frac{.03}{4}\right)^{32}$

$\approx 19{,}051.67$

Calculator:

1) $\frac{.03}{4}$ ↵ $+1$ ↵ $\boxed{x^y}$ 32 ↵

2) $\times 15{,}000$

$\boxed{\text{The balance after 8 years will be about \$19,051.67.}}$

Ex6) Write a compound interest function to $\boxed{\text{model}}$ \$17,000 invested at a rate of 3% compounded monthly. Then find the balance after 6 years.

Model: $A = 17{,}000\left(1+\frac{.03}{12}\right)^{12t}$

Calculator:

$t=6$: $A = 17{,}000\left(1+\frac{0.03}{12}\right)^{72}$

1) $\frac{0.03}{12}$ ↲ $+1$ ↲ $\boxed{x^y}$ 72

$\approx 20{,}348.12$

2) $\times$ 17000

$\boxed{\text{The balance after 6 years will be about \$20,348.12.}}$

Try) Write a compound interest function to model \$23,000 invested at a rate of 2% compounded quarterly. Then find the balance after 8 years.

Model : $A = 23{,}000\left(1 + \frac{0.02}{4}\right)^{4t}$

$t = 8$: $A = 23{,}000\left(1 + \frac{0.02}{4}\right)^{32}$

$\approx 26{,}979.99$

$$\boxed{\text{The balance after 8 years will be about \$26,979.99.}}$$